# Vectors - I

V Vijaykiran
M.Sc., B.Ed., NET

## Scalar quantities ( or scalars )

Quantities that have only magnitude and no direction associated with them are called scalar quantities.

Eg : distance, speed, temperature, density, volume, electric current …

## Vector quantities ( or vectors )

Quantities that have magnitude, direction and follow vectors laws of addition are called vector quantities.

Eg : displacement, velocity, acceleration, force, momentum …

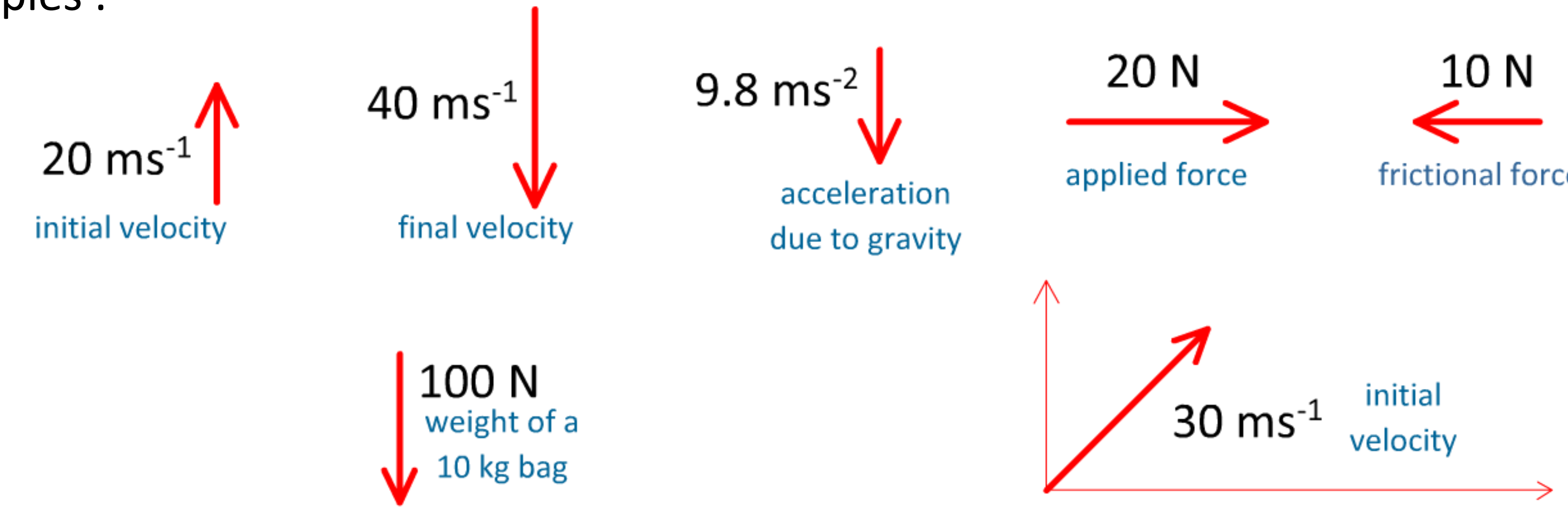
## Graphical representation of vectors

Graphically, a vector is represented by a directed line segment ( an arrow ).

Length of the arrow is an indication of magnitude of the vector.

Direction of arrowhead is in the direction of the vector .

Examples :




Graphical representations are useful only for a quick visualization / estimate of the phenomenon .

## Textual representation of vectors

Textually ( while writing ) a vector is represented with a bar over the symbol representing the vector quantity.

Textually ( in print ) a vector is represented with a bold face for the symbol representing the vector quantity

Examples :

| | In print | Hand written |
|---|---|---|
| displacement | $\boldsymbol{S}$ | $\overline{S}$ |
| velocity | $\boldsymbol{v}$ | $\overline{v}$ |
| acceleration | $\boldsymbol{a}$ | $\overline{a}$ |
| force | $\boldsymbol{F}$ | $\overline{F}$ |

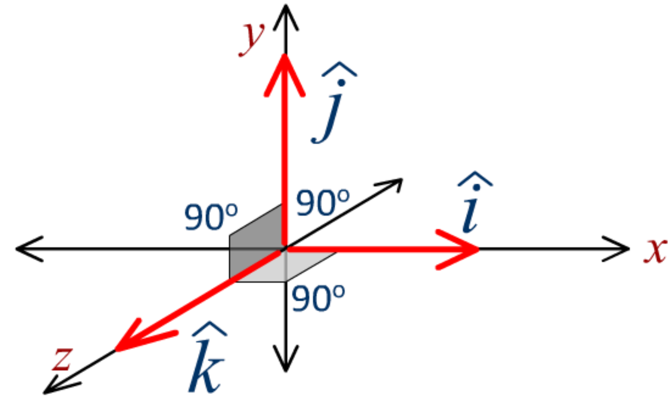
# Motion in a plane

## Types of vectors

- Unit vector : A vector of unit magnitude is called a unit vector.

  It is denoted by a cap on the symbol representing the vector.

  It is used to represent a specific direction.




- Position vector : A vector drawn fro the origin to the instantaneous position of the body is called a position vector.

  It is generally denoted by ***r.***

- Negative of a vector : Negative of a vector is a vector of same magnitude but opposite direction.

- Null vector : A vector of zero magnitude and unspecified direction is called a null vector.

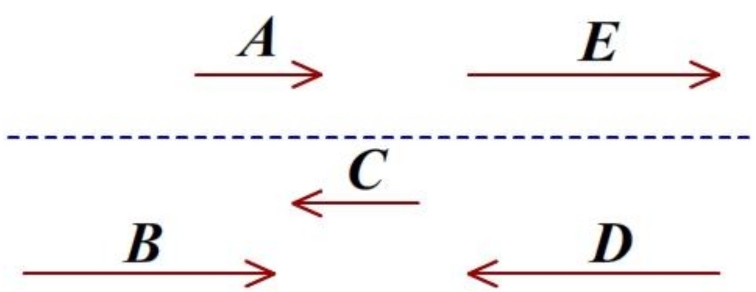
  It is denoted as **O**.

## Types of vectors

- Equal vectors : Two vectors are said to be equal if they have same magnitude and direction
- Parallel vectors : Two vectors are said to be parallel to each other they if they are parallel to the same reference line
- Like vectors : Two parallel vectors are said to be like if they are along the same direction.
- Unlike vectors : Two parallel vectors are said to be unlike if they are in opposite directions.

Example

- $\boldsymbol{A}$, $\boldsymbol{E}$ and $\boldsymbol{B}$ are like vectors
- $\boldsymbol{C}$ and $\boldsymbol{D}$ are like vectors
- $\boldsymbol{A}$ and $\boldsymbol{D}$ are unlike vectors
- ALL the vectors are parallel vectors



## Types of vectors

- Polar vector : A vector that reverses direction then the coordinate axes are reversed.

  Eg. Velocity, displacement etc.

- Axial vector : A vector that does not reverse sign when the coordinate axes are reversed. Such a vector is also called a pseudo-vector.

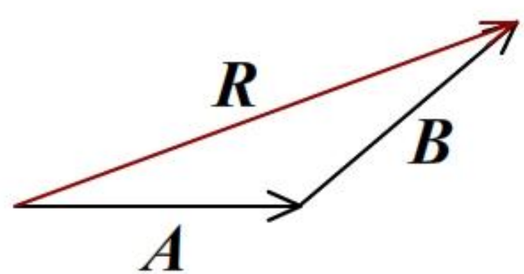
  Eg. Angular momentum etc.

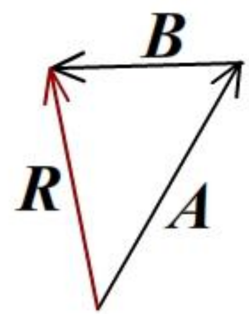
## Vector addition ( graphical methods )

**Triangle law of addition of vectors**

If two vectors represent two sides of a triangle taken in an order, both in magnitude and direction, then their resultant is given by the closing side of the triangle, both in magnitude and direction, taken in the reverse order.

Examples :



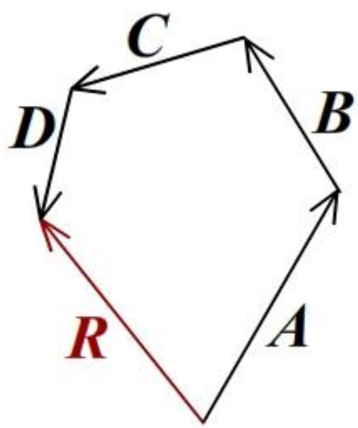



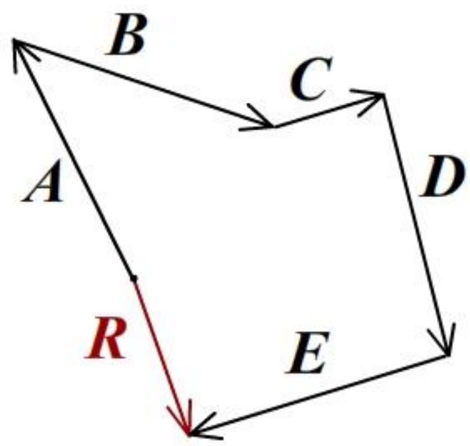
## Vector addition ( graphical methods )

**<u>Polygon law of addition of vectors</u>**

If a set of vectors represent adjacent sides of a polygon taken in an order, both in magnitude and direction, then their resultant is given, both in magnitude and direction, by the closing side of the polygon taken in the reverse order.

Examples :



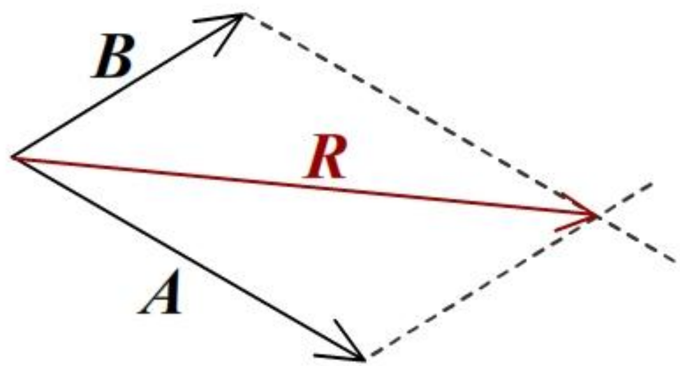
## Vector addition ( graphical methods )

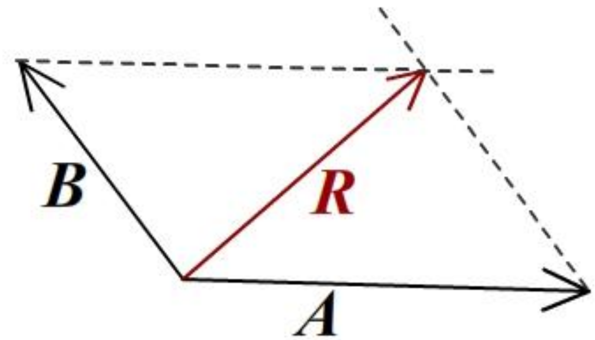
Click here for simulation

### Parallelogram law of addition of vectors

If two vectors represent adjacent sides of a parallelogram, both in magnitude and direction, then their resultant is given, both in magnitude and direction, by the diagonal of the parallelogram drawn from the same point.

Examples :

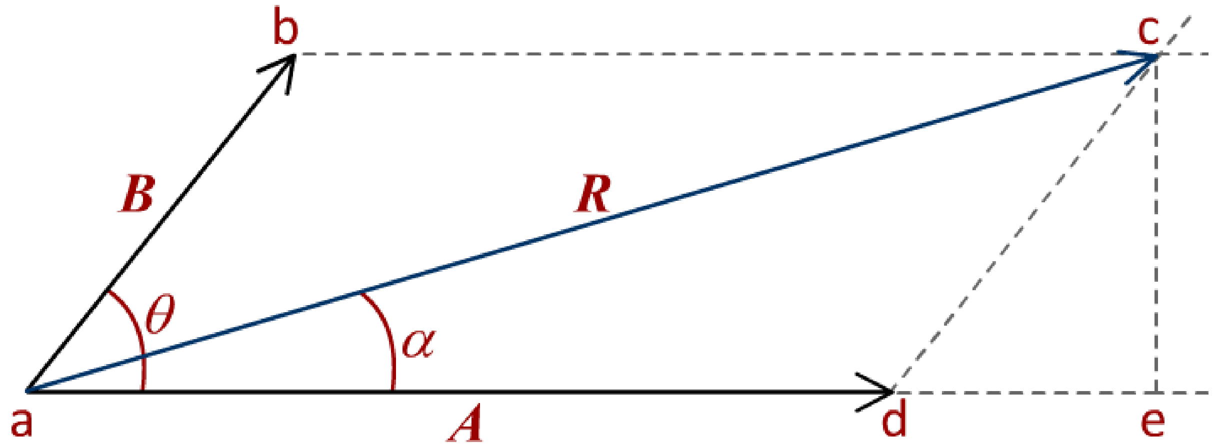


## Parallelogram law of addition of vectors ( magnitude )

Click here for simulation

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two vectors and $\theta$ be the angle between them. A line is drawn parallel to $\boldsymbol{A}$ and another line is drawn parallel to $\boldsymbol{B}$. A perpendicular is dropped from the point of intersection.




From $\Delta$ ace we get

$$ac^2 = ae^2 + ec^2$$

$$ac^2 = (ad + de)^2 + ec^2$$

$$ac^2 = ad^2 + de^2 + 2\,ad\,de + ec^2 \quad \text{i}$$

From $\Delta$ dce we get

$$\sin(\theta) = \frac{ec}{B} \Rightarrow ec = B\sin(\theta) \quad \text{ii}$$

$$\cos(\theta) = \frac{de}{B} \Rightarrow de = B\cos(\theta) \quad \text{iii}$$

Substituting eqs(ii) and (iii) in eq(i)

$$R^2 = A^2 + B^2\cos^2(\theta) + 2AB\cos(\theta) + B^2\sin^2(\theta)$$

$$R^2 = A^2 + B^2 + 2AB\cos(\theta)$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos(\theta)}}$$

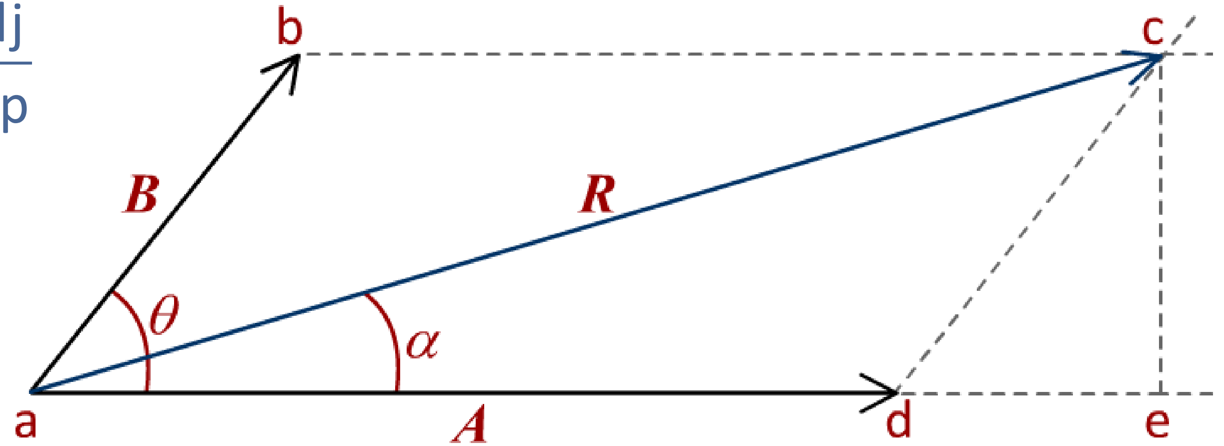
# Motion in a plane

## Parallelogram law of addition of vectors ( direction )

Click here for simulation

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} \qquad \cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$$




From Δ ace we get

$$\tan(\alpha) = \frac{\text{ec}}{\text{ae}}$$

$$\tan(\alpha) = \frac{\text{ec}}{(\text{ad} + \text{de})} \quad \text{i}$$

$$\text{ec} = B\sin(\theta) \quad \text{ii}$$

$$\text{de} = B\cos(\theta) \quad \text{iii}$$

Substituting eqs(ii) and (iii) in eq(i)

$$\tan(\alpha) = \frac{B\sin(\theta)}{A + B\cos(\theta)}$$

$$\boxed{\alpha = \tan^{-1}\left[\frac{B\sin(\theta)}{A + B\cos(\theta)}\right]}$$

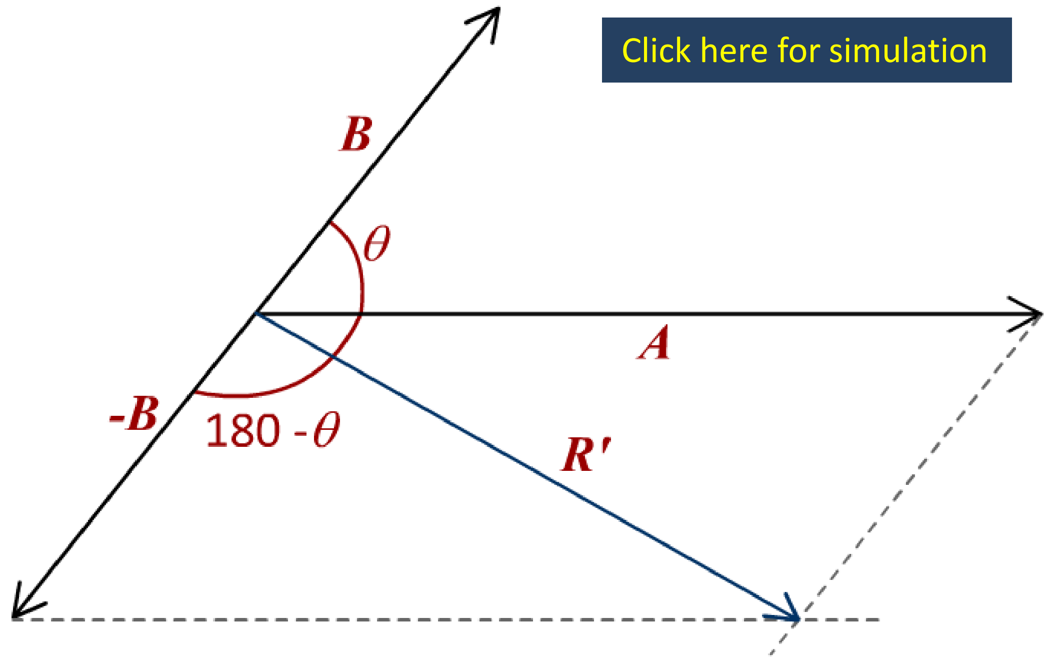
# Motion in a plane

## Subtraction of two vectors

$$\boldsymbol{R'} = \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$$

Take negative of the second vector and then add the vectors using parallelogram law !

Click here for simulation




$$R = \sqrt{A^2 + B^2 + 2AB\cos(\theta)}$$

$$R' = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

$$R' = \sqrt{A^2 + B^2 - 2AB\cos(\theta)}$$

$$\alpha' = \tan^{-1}\left[\frac{B\sin(180-\theta)}{A + B\cos(180-\theta)}\right]$$

$$\alpha' = \tan^{-1}\left[\frac{B\sin(\theta)}{A - B\cos(\theta)}\right]$$